

Title: The complexity of first-order and monadic second-order logic revisited

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Abstract: The model-checking problem for a logic L on a class C of structures asks whether a given L -sentence holds in a given structure in C . In this paper, we give super-exponential lower bounds for fixed-parameter tractable model-checking problems for first-order and monadic second-order logic.

We show that unless $\text{PTIME}=\text{NP}$, the model-checking problem for monadic second-order logic on finite words is not solvable in time $f(k) p(n)$, for any *elementary* function f and any polynomial p . Here k denotes the size of the input sentence and n the size of the input word. We prove the same result for first-order logic under a stronger complexity theoretic assumption from parameterized complexity theory.

Furthermore, we prove that the model-checking problems for first-order logic on structures of degree 2 and of bounded degree $d>2$ are not solvable in time $2^{\{2^{\{o(k)}\}} p(n)}$ (for degree 2) and $2^{\{2^{\{2^{\{o(k)}\}}\}} p(n)$ (for degree $d>2$), for any polynomial p , again under an assumption from parameterized complexity theory. We match these lower bounds by corresponding upper bounds.

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