

# **Title:** A Double Arity Hierarchy Theorem for Transitive Closure Logic

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**Abstract:** In this paper we prove that the  $k$ -ary fragment of transitive closure logic is not contained in the extension of the  $(k-1)$ -ary fragment of partial fixed point logic by all  $(2k-1)$ -ary generalized quantifiers. As a consequence, the arity hierarchies of all the familiar forms of fixed point logic are strict simultaneously with respect to the arity of the induction predicates and the arity of generalized quantifiers.

Although it is known that our theorem cannot be extended to the sublogic deterministic transitive closure logic, we show that an extension is possible when we close this logic under congruence.