Abstract: The local tree-width of a graph $G=(V,E)$ is the function $\text{ltw}^G: \mathbb{N} \rightarrow \mathbb{N}$ that associates with every natural number $r$ the maximal tree-width of an $r$-neighborhood in $G$. Our main graph theoretic result is a decomposition theorem for graphs with excluded minors that essentially says that such graphs can be decomposed into trees of graphs of bounded local tree-width. As an application of this theorem, we show that a number of combinatorial optimization problems, such as Minimum Vertex Cover, Minimum Dominating Set, and Maximum Independent Set have a polynomial time approximation scheme when restricted to a class of graphs with an excluded minor.